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Notes.

I.

Note on Partitions.

BY F. FRANKLIN, *Fellow of the Johns Hopkins University.*

IN a paper published in the Messenger of Mathematics (May, 1878), Prof. Sylvester has given a rule for abbreviating the calculation of $(w:i,j)-(w-1:i,j)$; where, to fix the ideas, let $(x:i,j)$ be regarded as the number of modes of composing x with j of the numbers $0, 1, 2, \ldots i$. The abbreviation consists in rejecting from the partitions of w all partitions whose highest number is not repeated and rejecting from the partitions of $w-1$ all partitions which do not contain i; the number of partitions thus rejected being shown to be the same in the two cases.

This becomes even more obvious if we convert the above (i, j) partitions into (j, i) partitions: that is, replace each of the above partitions by a corresponding one consisting of i of the numbers $0, 1, \ldots j$. This, as is well known, can be done by decomposing each number into a *column* of 1's and then recomposing by *rows*. Now, if we do this, it is plain that those partitions whose highest number was not repeated become partitions containing 1; and that those partitions which did not contain i become partitions having less than the full number of parts, or, in other words, partitions containing 0. So that Prof. Sylvester's abbreviation is equivalent to rejecting from the partitions of w those partitions which contain 1, and from the partitions of $w-1$ those which contain 0. And it is plain that the number of partitions of w which contain 1 is equal to the number of partitions of $w-1$ which contain 0; for the two sets of partitions are interchanged by the interchange of 0 and 1.

Obviously, instead of rejecting the partitions of w which contain 1 and those of $w-1$ which contain 0, we may reject the partitions of w which contain m (where m is any one of the numbers $1, 2, \ldots i$ (or j)) and those of $w-1$ which contain $m-1$; the reason being the same as above.

It may also be observed that the theorem in this form is easily obtained from the generating function. For $(w:i,j)-(w-1:i,j)$ is the coefficient of $a^w x^j$ in the development of

$$\frac{1-a}{(1-x)(1-ax)(1-a^2x)\ldots(1-a^ix)},$$

the numerator of which fraction may be written

$$1-a^m x-a\,(1-a^{m-1}x),$$

so that $(w:i,j)-(w-1:i,j)$ is the difference between the coefficient of $a^w x^j$ in the development of

$$\frac{1-a^m x}{(1-x)(1-ax)\ldots(1-a^ix)};$$

and the coefficient of $a^{w-1}x^j$ in the development of

$$\frac{1-a^{m-1}x}{(1-x)(1-ax)\ldots(1-a^ix)};$$

and obviously if m is one of the numbers $1, 2, \ldots i$, these coefficients are, respectively, the number of the (i,j) partitions of w which do not contain m, and the number of the (i,j) partitions of $w-1$ which do not contain $m-1$: wherefore, the partitions of w which do contain m and those of $w-1$ which do contain $m-1$ may be simultaneously rejected.

II.

Some General Formulæ for Integrals of Irrational Functions.

By W. I. Stringham, *Fellow of the Johns Hopkins University.*

LET $\operatorname{Sh} z=\frac{ax+c}{\sqrt{b^2-c^2}}$, $\operatorname{Ch} z=\frac{aX+c}{\sqrt{b^2-c^2}}$. Since $\operatorname{Ch}^2 z-\operatorname{Sh}^2 z=1$, therefore

$$aX+c=\sqrt{a^2x^2+2acx+b^2}.$$

Let $\frac{c}{a}=h$, and $\frac{b^2-c^2}{a^2}=e^2$. Then

$$\int(x+h)^m(X+h)^n\,dx=e^{m+n+1}\int\operatorname{Sh}^m z\ \operatorname{Ch}^{n+1}z\,.\,dz.$$